

## Friday 4      Doppler Effect

For sound waves, the relation between frequency  $f$  and wavelength  $\lambda$  of a harmonic wave is determined by the sound speed in a medium  $S$ :  $S = f \cdot \lambda$ . Wave speed equals frequency times wavelength. This relation always holds if you are "at rest" (that means not moving) relative to the material that the wave is traveling in. In physics, we say that all the objects that are not moving relative to you are in your "frame of reference". **The relation between frequency and wavelength given above only holds in the frame of reference of the air** (the material that the wave traveling in). If either the source of a sound is moving or if the receiver is moving (or both) relative to the air then you will need to make a correction called the Doppler effect. Actually, there are two separate effects called the Doppler effect: one where the source moves and the other when the receiver moves and we will look at each in turn.

### 1) The moving source Doppler effect.

If a sound source (for example an automobile) is moving then the frequency of the sound waves it makes will be different than the frequency of the source. Motion of the source causes the frequency of the sound to be different in different directions. In the direction that the source is moving, the frequency is increased. If a source is moving away from you, then the frequency you hear is reduced. If  $v_s$  is the speed of the source (we will call this a positive number if it is moving toward the receiver),  $f_{source}$  is the frequency of the sound source, and  $f_{air}$  is the frequency of the sound heard by the receiver in the frame of reference of the air:

$$\frac{f_{source}}{f_{air}} = 1 - \frac{v_s}{S}$$

### 2) The moving receiver Doppler effect.

If a receiver (for example a microphone) is moving then the frequency it receives will be affected by its direction of motion. If the receiver moves toward the source, the frequency it hears will be increased, if it moves away from the source then its frequency will be diminished. If  $v_o$  is the speed of the receiver (we will call this a positive number if it is moving toward the source),  $f_{air}$  is the frequency of the wave in the frame of reference of the air, and  $f_{receiver}$  is the frequency heard by the receiver:

$$\frac{f_{receiver}}{f_{air}} = 1 + \frac{v_o}{S}$$

NOTE, from either effect, the frequency increases if either the source or the receiver is in motion toward the other.

Friday 4

Name: \_\_\_\_\_

Measuring speed using the Doppler effect.

Doppler radar is used to catch speeders. Here we will use the Doppler effect for sound to determine the speed of a moving source of sound.

A spectrogram is a picture that shows the analysis of sound in terms of the frequencies that are present at each moment in time. For example a sound of constant amplitude and frequency shows up as a bright horizontal band on the plot. Color is used to indicate intensity in this plot. The horizontal axis tells you the time when the sound occurred and the vertical axis tells you the frequency. Many sounds consist of multiple frequencies at the same time.

A source in motion will result in a shift in the frequency due to the Doppler effect.

- 1) What is the frequency of the whistle when it is not moving? This is the frequency of the source in its own frame of reference which we call  $f_{source}$ .

**From the graph it is about 3200 Hz**

- 2) What is the maximum frequency that the whistle is Doppler shifted to when it was twirled around? This is the frequency of the sound in the frame of reference of the air when the whistle is at its maximum speed toward the microphone.  $f_{air}$

**From the last graph we can see that it is about 3300 Hz**

- 3) What is the ratio of the frequencies:  $\frac{f_{source}}{f_{air}}$  ?

**3200/3300=0.97**

With two steps of algebra we can use the formula for the Doppler shift with a moving source to determine the speed of the source (we will take the speed of sound to be  $v=340$  m/s).

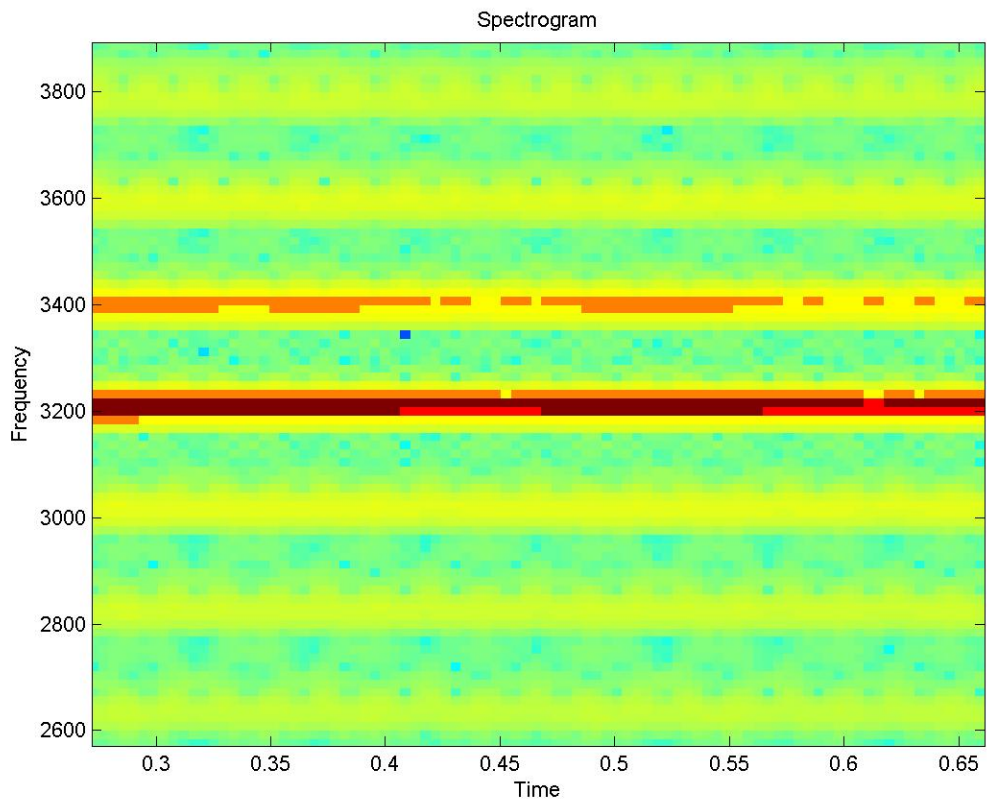
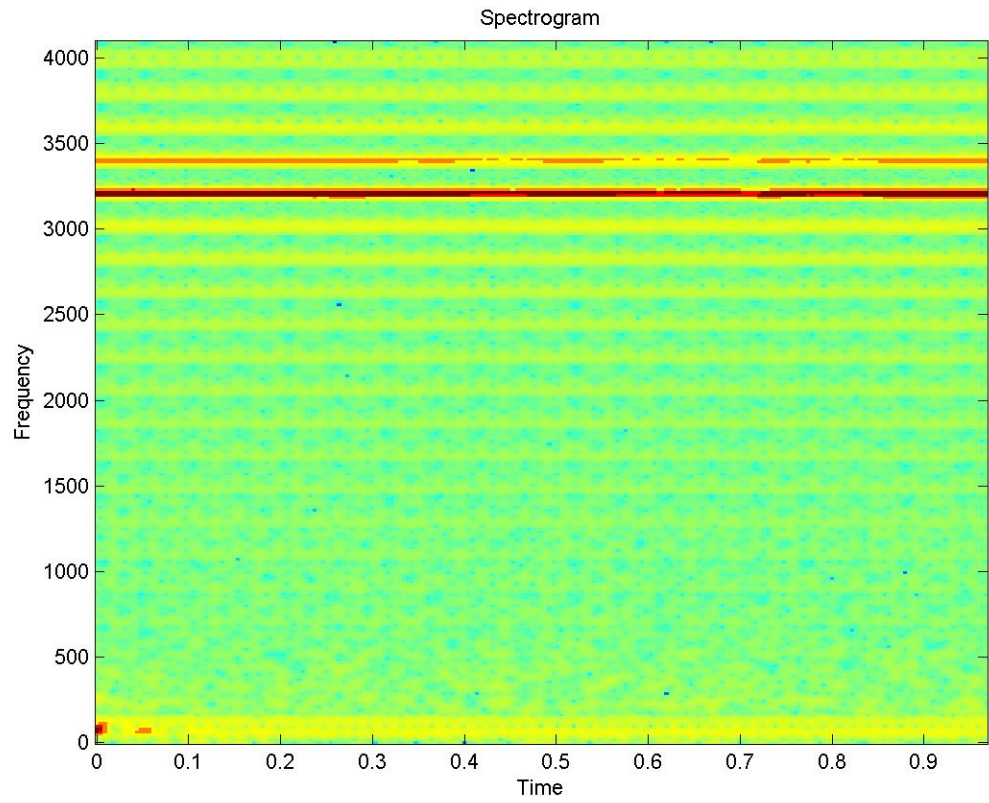
If we start with  $\frac{f_{source}}{f_{air}} = 1 - \frac{v_s}{S}$  then  $\frac{v_s}{S} = 1 - \frac{f_{source}}{f_{air}}$  and  $v_s = S(1 - \frac{f_{source}}{f_{air}})$ .

- 4) What is the maximum speed of the swinging whistle?

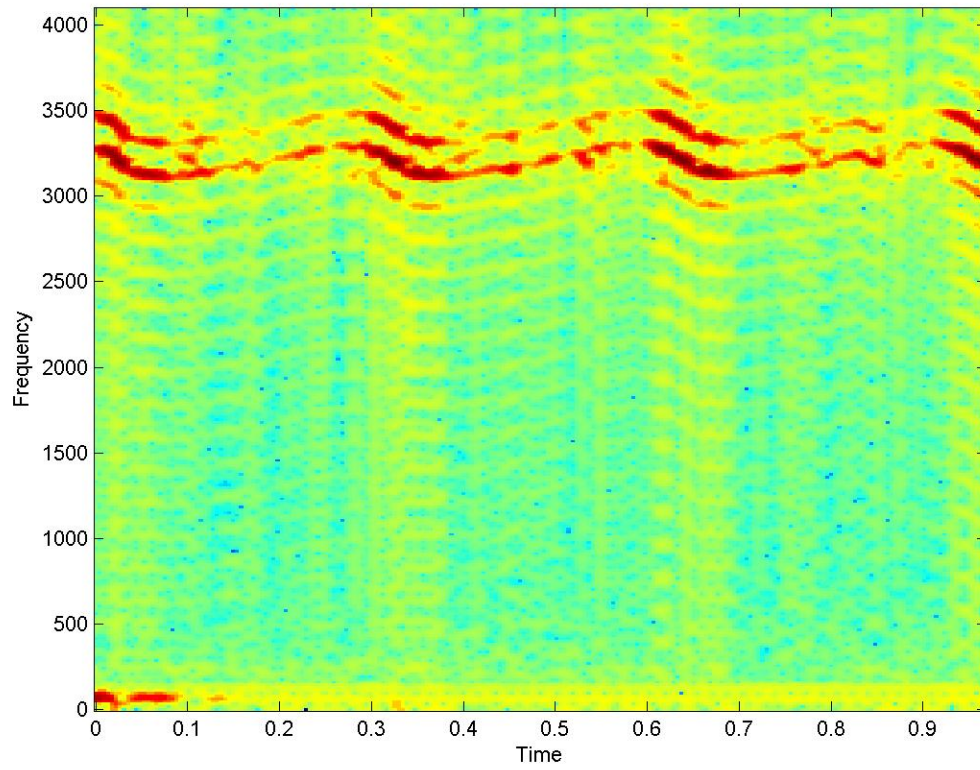
**$v_s = 340 * (1 - 0.97) \text{ m/s} = 10.3 \text{ m/s}$**

- 5) What is the length of the string R ?

**We can see the "orbital" period from the graph (T=0.3 s). Given  $v_s = 2\pi R/T$  we can find  $R = v_s * T / (2\pi) = (10.3 \text{ m/s}) * (0.3 \text{ s}) / (6.28) = 0.5 \text{ m}$**



Moving Whistle Spectrogram



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